

Successive - Differentiation

Chapter: - 1

① Theorem: -

Q.N. → find nth diff. of $\sin(ax+b)$

or, If $y = \sin(ax+b)$ then find y_n

Ans. → Let $y = \sin(ax+b)$

$$y_1 = a \cos(ax+b)$$

$$y_2 = a^2 \sin\left(\frac{\pi}{2} + ax+b\right)$$

$$y_3 = a^3 \cos\left(2\frac{\pi}{2} + ax+b\right)$$

$$y_4 = a^4 \sin\left(3\frac{\pi}{2} + ax+b\right)$$

$$y_n = a^n \sin\left(m\frac{\pi}{2} + ax+b\right)$$

② Q.N. → If $y = \cos(ax+b)$

Ans. → Let $y = \cos(ax+b)$

$$y_1 = -a \sin(ax+b)$$

$$y_2 = a^2 \cos\left(\frac{\pi}{2} + ax+b\right)$$

$$y_3 = -a^3 \sin\left(2\frac{\pi}{2} + ax+b\right)$$

$$y_4 = a^4 \cos\left(3\frac{\pi}{2} + ax+b\right)$$

$$y_n = a^n \cos\left(m\frac{\pi}{2} + ax+b\right)$$

③ Q.N. → If $y = e^{ax} \sin x$, then find y_n

$$\text{Ans.} \rightarrow \therefore y = e^{ax} \sin bx$$

∴ b. S. w. r. t. x , we have,

$$y_1 = a \cdot e^{ax} \sin bx + e^{ax} \times \cos bx \times b$$

$$= e^{ax} (a \sin bx + b \cos bx)$$

$$\text{Let } a = r \cos \theta \quad \text{--- (1)}$$

$$b = r \sin \theta \quad \text{--- (2)}$$

Squaring and adding (1) and (2), we have

$$r^2 = a^2 + b^2 \Rightarrow r = (a^2 + b^2)^{1/2}$$

$$\text{(2)} \div \text{(1)}$$

$$\frac{r \sin \theta}{r \cos \theta} = \frac{b}{a} \quad \therefore \tan \theta = \frac{b}{a} \quad \text{or } \theta = \tan^{-1} \frac{b}{a}$$

$$y_1 = e^{ax} (r \cos \theta \sin bx + r \sin \theta \cos bx)$$

$$= r e^{ax} (\sin bx \cos \theta + \cos bx \sin \theta)$$

$$y_1 = r e^{ax} \sin (bx + \theta)$$

$$y_2 = r^2 e^{ax} \sin (bx + 2\theta)$$

$$y_3 = r^3 e^{ax} \sin (bx + 3\theta)$$

$$y_m = r^m e^{ax} \sin (bx + m\theta)$$

Putting the value of r and θ , we have

$$y_m = (a^2 + b^2)^{m/2} e^{ax} \sin (bx + m \cdot \tan^{-1} \frac{b}{a})$$

Ans. $\rightarrow \therefore y = e^{ax} \cos bx$

D. b. S. w. r. t. x , we have

$$y_1 = a \cdot e^{ax} \cos bx + e^{ax} x - \sin bx \cdot b$$

$$= e^{ax} (a \cos bx - b \sin bx)$$

Let $a = r \cos \theta$ — (1)

$b = r \sin \theta$ — (2)

Squaring and adding (1) and (2), we have,

$$r^2 = a^2 + b^2 \quad r = (a^2 + b^2)^{1/2}$$

(2) \div (1)

$$\frac{r \sin \theta}{r \cos \theta} = \frac{b}{a} \quad \therefore \tan \theta = \frac{b}{a}, \quad \theta = \tan^{-1} \frac{b}{a}$$

$$y_1 = e^{ax} (r \cos \theta \cdot \cos bx - r \sin \theta \cdot \sin bx)$$

$$y_1 = r e^{ax} \cos (bx + \theta)$$

$$y_2 = r^2 e^{ax} \cos (bx + 2\theta)$$

$$y_3 = r^3 e^{ax} \cos (bx + 3\theta)$$

$$y_m = r^m e^{ax} \cos (bx + m\theta)$$

Putting the value of r and θ , we have

$$y_m = (a^2 + b^2)^{m/2} e^{ax} \cos (bx + m \cdot \tan^{-1} \frac{b}{a})$$

Q. 10. \rightarrow If $y = e^{ax} \sin (bx + c)$, then find y_m .

Ans. $\therefore y = e^{ax} \sin(bx+c)$

D. b. S. w. r. t. x , we have

$$\begin{aligned}
 y_1 &= a \cdot e^{ax} \sin(bx+c) + e^{ax} \times \cos(bx+c) \times ab \\
 &= a e^{ax} \sin(bx+c) + ab e^{ax} \cos(bx+c) \\
 &= e^{ax} (a \sin(bx+c) + b \cos(bx+c))
 \end{aligned}$$

Let $a = r \cos \theta$ — (1)

$b = r \sin \theta$ — (2)

Squaring and adding (1) and (2), we have,

$$r^2 = a^2 + b^2, \quad r = (a^2 + b^2)^{\frac{1}{2}}$$

(2) \div (1)

$$\frac{r \sin \theta}{r \cos \theta} = \frac{b}{a} \quad \therefore \tan \theta = \frac{b}{a} \quad \therefore \theta = \tan^{-1} \frac{b}{a}$$

$$y_1 = e^{ax} (r \cos \theta \sin(bx+c) + r \sin \theta \cos(bx+c))$$

$$= r e^{ax} (\sin(bx+c) \cos \theta + \cos(bx+c) \sin \theta)$$

$$y_1 = r e^{ax} \sin(bx+c+\theta)$$

$$y_2 = r^2 e^{ax} \sin(bx+c+2\theta)$$

$$y_3 = r^3 e^{ax} \sin(bx+c+3\theta)$$

$$y_n = r^n e^{ax} \sin(bx+c+n\theta)$$

Putting the value of r and θ , we have

$$y_n = (a^2 + b^2)^{\frac{n}{2}} e^{ax} \sin(bx+c+n \tan^{-1} \frac{b}{a})$$

⑥ QN. → Find y_m when $y = e^{ax} \cos(bx+c)$.

Ans. → $y = e^{ax} \cos(bx+c)$

D. b. S. w. r. t. x , we have

$$y_1 = ax e^{ax} \cos(bx+c) + e^{ax} \{-\sin(bx+c)\} \times b$$

$$y_1 = a e^{ax} \cos(bx+c) - b e^{ax} \sin(bx+c)$$

$$= e^{ax} (a \cos(bx+c) - b \sin(bx+c))$$

Let θ

$$a = r \cos \theta \quad \text{--- (1)}$$

$$b = r \sin \theta \quad \text{--- (2)}$$

Squaring and adding (1) and (2) we have

$$r^2 = a^2 + b^2, \quad r = (a^2 + b^2)^{1/2}$$

$$(2) \div (1)$$

$$\frac{r \sin \theta}{r \cos \theta} = \frac{b}{a}$$

$$= \tan \theta = \frac{b}{a} \quad \therefore \theta = \tan^{-1} \frac{b}{a}$$

$$y_1 = e^{ax} [r \cos \theta \cdot \cos(bx+c) - r \sin \theta \cdot \sin(bx+c)]$$

$$= r e^{ax} [\cos(bx+c) \cdot \cos \theta - \sin(bx+c) \cdot \sin \theta]$$

$$y_1 = r e^{ax} \cos(bx+c+\theta)$$

$$y_2 = r^2 e^{2ax} \cos(bx+c+2\theta)$$

$$y_3 = r^3 e^{3ax} \cos(bx+c+3\theta)$$

$$y_m = r^m e^{m ax} \cos(bx+c+n\theta)$$

Putting the value of r and θ , we have

$$y_m = (a^2 + b^2)^{m/2} e^{m ax} \cos(bx+c+n \cdot \tan^{-1} \frac{b}{a})$$