Successive - Differentiation

Theorem:-

① QN.→ find nth diff. of $\sin(ax+b)$

or, If $y=\sin(ax+b)$ then find y_n

Ans.→ Let $y=\sin(ax+b)$

$$y_1 = a\cos(ax+b)$$

$$y_1 = a\sin\left(\frac{\pi}{2}+ax+b\right)$$

$$y_2 = a^2\sin\left(2\frac{\pi}{2}+ax+b\right)$$

$$y_3 = a^3\sin\left(3\frac{\pi}{2}+ax+b\right)$$

$$y_n = a^n\sin\left(n\frac{\pi}{2}+ax+b\right)$$

② QN.→ If $y=\cos(ax+b)$

Ans.→ Let $y=\cos(ax+b)$

$$y_1 = a\times -\sin(ax+b)$$

$$y_1 = a\cos\left(\frac{\pi}{2}+ax+b\right)$$

$$y_2 = a^2\cos\left(2\frac{\pi}{2}+ax+b\right)$$

$$y_3 = a^3\cos\left(3\frac{\pi}{2}+ax+b\right)$$

$$y_n = a^n\cos\left(n\frac{\pi}{2}+ax+b\right)$$

③ QN.→ If $y=e^{ax}\sin bx$, then find y_n.

Ans. → ∵ $y = e^{ax} \sin bx$

D.b.S. w.r.t. x, we have,

$y_1 = a\, e^{ax} \sin bx + e^{ax} \times \cos bx \times b$

$= e^{ax}(a \sin bx + b \cos bx)$

Let $a = r\cos\theta$ — ①

$b = r\sin\theta$ — ②

squaring and adding ① and ②, we have

$r^2 = a^2 + b^2 \Rightarrow r = (a^2+b^2)^{\frac{1}{2}}$

② ÷ ①

$\frac{r\sin\theta}{r\cos\theta} = \frac{b}{a}$ ∴ $\tan\theta = \frac{b}{a}$ or, $\theta = \tan^{-1}\frac{b}{a}$

$y_1 = e^{ax}(r\cos\theta . \sin bx + r\sin\theta . \cos bx)$

$= r e^{ax}(\sin bx . \cos\theta + \cos bx . \sin\theta)$

$y_1 = r\, e^{ax} \sin(bx+\theta)$

$y_2 = r^2 e^{ax} \sin(bx+2\theta)$

$y_3 = r^3 e^{ax} \sin(bx+3\theta)$

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$y_n = r^n e^{ax} \sin(bx+n\theta)$

Putting the value of r and θ, we have

$y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax} \sin\left(bx + n . \tan^{-1}\frac{b}{a}\right)$.

Ans.→ ∵ $y = e^{ax}\cos bx$

D.b.S.w.r.t. x, we have

$y_1 = a \cdot e^{ax}\cos bx + e^{ax} \times -\sin bx \times b$

$= e^{ax}(a\cos bx - b\sin bx)$

Let $a = r\cos\theta$ — ①

$b = r\sin\theta$ — ②

Squaring and adding ① and ②, we have,

$r^2 = a^2 + b^2$ ∴ $r = (a^2+b^2)^{1/2}$

② ÷ ①

$\frac{r\sin\theta}{r\cos\theta} = \frac{b}{a}$ ∴ $\tan\theta = \frac{b}{a}$ ∴ $\theta = \tan^{-1}\frac{b}{a}$

$y_1 = e^{ax}(r\cos\theta \cdot \cos bx - r\sin\theta \cdot \sin bx)$

$y_1 = r e^{ax}\cos(bx+\theta)$

$y_2 = r^2 e^{ax}\cos(bx+2\theta)$

$y_3 = r^3 e^{ax}\cos(bx+3\theta)$

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$y_n = r^n e^{ax}\cos(bx+n\theta)$

Putting the value of r and θ, we have

$y_n = (a^2+b^2)^{n/2} e^{ax}\cos\left(bx + n\cdot\tan^{-1}\frac{b}{a}\right)$.

⑤ QN.→ If $y = e^{ax}\sin(bx+c)$, then find y_n

Ans.→ ∵ $y = e^{ax}\sin(bx+c)$

D.b.S.w.r.t. x, we have

$$y_1 = a\cdot e^{ax}\sin(bx+c) + e^{ax}\times\cos(bx+c)\times b$$

$$= a\,e^{ax}\sin(bx+c) + b\,e^{ax}\cos(bx+c)$$

$$= e^{ax}\left(a\sin(bx+c) + b\cos(bx+c)\right)$$

Let $a = r\cos\theta$ — (1)

$b = r\sin\theta$ — (2)

Squaring and adding (1) and (2), we have,

$r^2 = a^2 + b^2$, $r = (a^2+b^2)^{1/2}$

(2) ÷ (1)

$$\frac{r\sin\theta}{r\cos\theta} = \frac{b}{a} \quad \therefore \tan\theta = \frac{b}{a} \quad \therefore \theta = \tan^{-1}\frac{b}{a}$$

$$y_1 = e^{ax}\left(r\cos\theta\times\sin(bx+c) + r\sin\theta\cos(bx+c)\right)$$

$$= r\,e^{ax}\left(\sin(bx+c)\cdot\cos\theta + \cos(bx+c)\cdot\sin\theta\right)$$

$$y_1 = r\,e^{ax}\sin(bx+c+\theta)$$

$$y_2 = r^2 e^{ax}\sin(bx+c+2\theta)$$

$$y_3 = r^3 e^{ax}\sin(bx+c+3\theta)$$

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$$y_n = r^n e^{ax}\sin(bx+c+n\theta)$$

Putting the value of r and θ, we have

$$y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax}\sin\left(bx+c+n\tan^{-1}\frac{b}{a}\right)$$

⑥ QN.→ Find y_n when $y = e^{ax}\cos(bx+c)$.

Ans.→ $\because y = e^{ax}\cos(bx+c)$

D.b.S.w.r.t. x, we have

$y_1 = a \times e^{ax}\cos(bx+c) + e^{ax}\{-\sin(bx+c)\times b$

$y_1 = a e^{ax}\cos(bx+c) - b e^{ax}\sin(bx+c)$

$= e^{ax}(a\cos(bx+c) - b\sin(bx+c))$

Let,

$a = r\cos\theta$ — ①

$b = r\sin\theta$ — ②

squaring and adding ① and ② we have

$r^2 = a^2 + b^2$, $r = (a^2+b^2)^{1/2}$

② ÷ ①

$\frac{r\sin\theta}{r\cos\theta} = \frac{b}{a}$

$= \tan\theta = \frac{b}{a}$ $\therefore \theta = \tan^{-1}\frac{b}{a}$

$y_1 = e^{ax}\left[r\cos\theta\cdot\cos(bx+c) - r\sin\theta\cdot\sin(bx+c)\right]$

$= r e^{ax}\left[\cos(bx+c)\cdot\cos\theta - \sin(bx+c)\cdot\sin\theta\right]$

$y_1 = r e^{ax}\cos(bx+c+\theta)$

$y_2 = r^2 e^{ax}\cos(bx+c+2\theta)$

$y_3 = r^3 e^{ax}\cos(bx+c+3\theta)$

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$y_n = r^n e^{ax}\cos(bx+c+n\theta)$

putting the value of r and θ, we have

$y_n = (a^2+b^2)^{n/2} e^{ax}\cos\left(bx+c+n\cdot\tan^{-1}\frac{b}{a}\right)$